Symmetry of Combinations

Let's see the algebraic proof of the notion that selecting *p*-many elements out of a set of *n* is the same as omitting *n*-*p* many elements.

For starters, recall the combination formula:

$$C(n,p) = \frac{n!}{(n-p)!\,p!}$$

.

If we plug in n - p for p, we get the following:

$$C(n, n-p) = \frac{n!}{(n-(n-p))! (n-p)!} = \frac{n!}{(n-n+p)! (n-p)!} = \frac{n!}{p! (n-p)!} = \frac{n!}{(n-p)! p!} = C(n, p)$$

Therefore, we can conclude that C(n, p) = C(n, n - p).

