# BASIC PROBABILITY

## The likelihood of an event occurring

$$P(X) = \frac{\text{preferred outcomes}}{\text{sample space}}$$

$$P(A, B) = P(A) + P(B)$$

## EXPECTED VALUES

| Trial | Experiment | Experimental Probability | Expected Value |
|---|---|---|---|
| Observing an event and record the outcome | Collection of one or multiple trials | Probability of an event based on the experiment | Specific outcome we expect to occur when we run an experiment |

**Categorical** $E(X) = n \cdot p$

**Numeric** $E(X) = \sum_{i=1}^{n} n_i \cdot p_i$

## FREQUENCY

| | |
|---|---|
| **Probability frequency distribution** | Collection of the probabilities for each possible outcome of an event. |
| **Why?** | To try and predict future events when the expected value is unattainable. |
| **Frequency** | Number of times a given value or outcome appears in the sample space. |
| **Frequency distribution table** | Table that matches each distinct outcome in the sample space to its associated frequency. |
| **How?** | By dividing every frequency by the size of the sample space. |

## COMPLEMENTS

Everything an event is **not**

A’ = Not A

P(A’) = 1- P(A)

A + A’ = Sample space

# COMBINATORICS

## The likelihood of an event occurring

$$P(X) = \frac{\text{preferred outcomes}}{\text{sample space}}$$

$$P(A, B) = P(A) + P(B)$$

Without repetition

With repetition

## PERMUTATIONS (arrange)

$$P_n = P_n^n = n!$$

How many ways are to arrange 3 letters a, b, c?

How many ways are to arrange 2 letters a and 2 letters b?

$$P_{n_1,\ldots,n_k} = \frac{(\sum n_i)!}{\prod n_i!}$$

## VARIATIONS (pick and arrange)

$$V_p^n = P_p^n = \frac{n!}{(n-p)!}$$

Hoy many words of 2 different letters can you make with 4 letters a, b, c, d?

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$$\bar{V}_p^n = n^p$$

## COMBINATIONS (pick)

$$C_p^n = \frac{n!}{p!\,(n-p)!}$$

Hoy many ways are there to pick 2 different letters out of 4 letters a, b, c, d?

Hoy many ways are there to pick 2 letters out of 4 letters a, b, c, d?

$$\bar{C}_p^n = \mathrm{C}_{\mathrm{p}}^{\mathrm{n+p-1}}$$

## FACTORIALS

0! = 1
If n < 0, n! doesn't exist

If n > 0, n > k

$$(n + k)! = n! \cdot (n+1) \cdot \ldots \cdot (n+k)$$

$$(n-k)! = \frac{n!}{(n-k+1) \cdot \ldots \cdot n}$$

$$\frac{n!}{k!} = (k+1) \cdot \ldots \cdot n$$

# BAYESIAN INFERENCE

$x \in A$, $x \notin A$: element x is a part of set A (x NOT in A)

A $\ni x$: set A contains element x

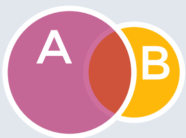
$\forall x$: for all/any x

$A \subseteq B$: A is a subset of B


**Mutually exclusive**

$A \cap B = \emptyset$

All complements are mutually exclusive, but not all mutually exclusive sets are complements




**Intersect**

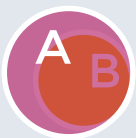
$A \cap B$

Satisfies all the events simultaneously

**Union**

$A \cup B = A + B - A \cap B$

Satisfies at least one of the events




**Completely overlap**

## Independent Events

$P(A|B) = P(A)$

Theoretically probability remains unaffected by other events

## Dependent Events

$P(A|B) \neq P(A)$

Probabilities of dependent events vary as conditions change

## Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- B has occurred
- Only elements of the intersection can satisfy A
- P(A|B) not the same meaning as P(B|A)

## Law of Total Probability

$A = B_1 + \ldots + B_n$

$P(A) = P(A|B_1) \cdot P(B_1) + \ldots + P(A|B_n) \cdot P(B_n)$

## Additive Law

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

## Multiplication Rule

$P(A \cap B) = P(A|B) \cdot P(B)$

## Bayes' Law

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

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# DISTRIBUTIONS

**Show the possible values a random variable can take and how frequently they occur.**

- Y actual outcome
- Y one of the possible outcomes
- P(Y = y) = p(y)
- Probability function: function that assigns a probability to each distinct outcome in the sample space

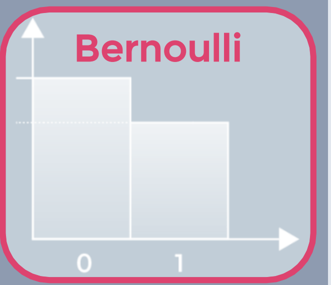
| | Population | Sample |
|---|---|---|
| **Mean** | $\mu$ | $\bar{x}$ |
| **Variance** | $\sigma^2$ | $s^2$ |
| **Standard Deviation** | $\sigma$ | s |

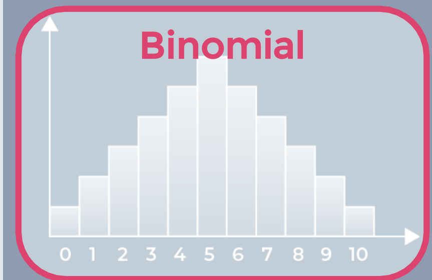
## DISCRETE

- Finite number of outcomes
- Can add up individual value to determine the probability of an interval
- Expressed with table, graph or piecewise function
- Expected values might be unattainable


- Y ~ U(a, b)
- Y ~ U(a) for categorical
- Outcomes are equally likely
- No predictive power

- Y ~ Bern(p)
- 1 trial, 2 possible outcomes
- E(Y) = p
- $\sigma^2$ (Y) = p • (1-p)




- Y ~ B(n, p)
- Measures the p 1 of the possible outcomes over n trials
- $P(Y=y) = p(y) = C(y, n) \cdot p^y \cdot (1-p)^{n-y}$
- E(Y) = n • p
- $\sigma^2$ (Y) = n • p • (1-p)

- Y ~ Po($\lambda$)
- Measures the frequency over an interval of time or distance ($\lambda \geq 0$)
- $P(Y=y) = p(y) = \dfrac{\lambda^y}{y!\, e^{-\lambda}}$
- E(Y) = $\lambda$
- $\sigma^2$ (Y) = $\lambda$

# DISTRIBUTIONS

- PDF: Probability Density Function
- CDF: Cummulative Density Function

## CONTINUOUS

- Infinitely many consecutive possible values
- Cannot add up individual value to determine the probability of an interval
- Expressed with graph or continuous function
- $P(Y=y) = p(y) = 0$ for any individual value y ($P(Y< y) = P( Y \leq y)$

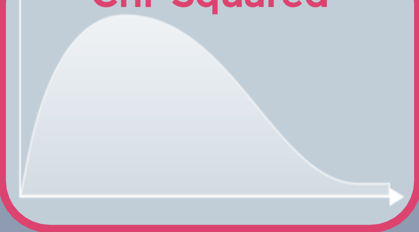
### Normal

- $Y \sim N(\mu, \sigma^2)$
- $E(Y) = \mu$
- $\sigma^2(Y) = \sigma^2$
- 68% of all ist values fall in the interval $(\mu - \sigma,\ \mu + \sigma)$

### Students' T

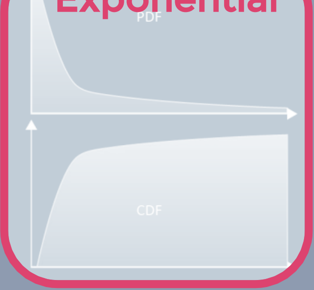
- $Y \sim t(k)$
- Small sample size approximation of a Normal (accounts for extreme values better)
- If k>1: $E(Y) = \mu$ and $\sigma^2 (Y) = s2 \cdot k / (k-2)$

### Chi-Squared



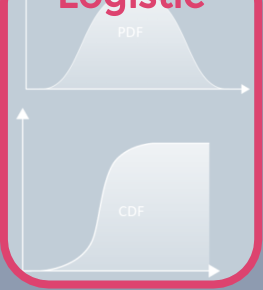
- $Y \sim \chi 2(\lambda)$
- Square of the t-distribution
- $E(Y) = k$
- $\sigma^2 (Y) = 2k$

### Exponential




- $Y \sim Exp(\lambda)$
- $E(Y) = 1/\lambda$
- $\sigma^2 (Y) = 1/\lambda^2$

### Logistic




- $Y \sim Logistic(\mu, s)$
- Continous variable inputs and binary outcome
- CDF ↑ when values near the mean
- ↓ *s*, the quicker it reaches values close to 1
- $E(Y) = \mu$
- $\sigma^2 (Y) = s^2 \cdot n^2 / 3$