Course notes: Regression Analysis

What is linear regression?

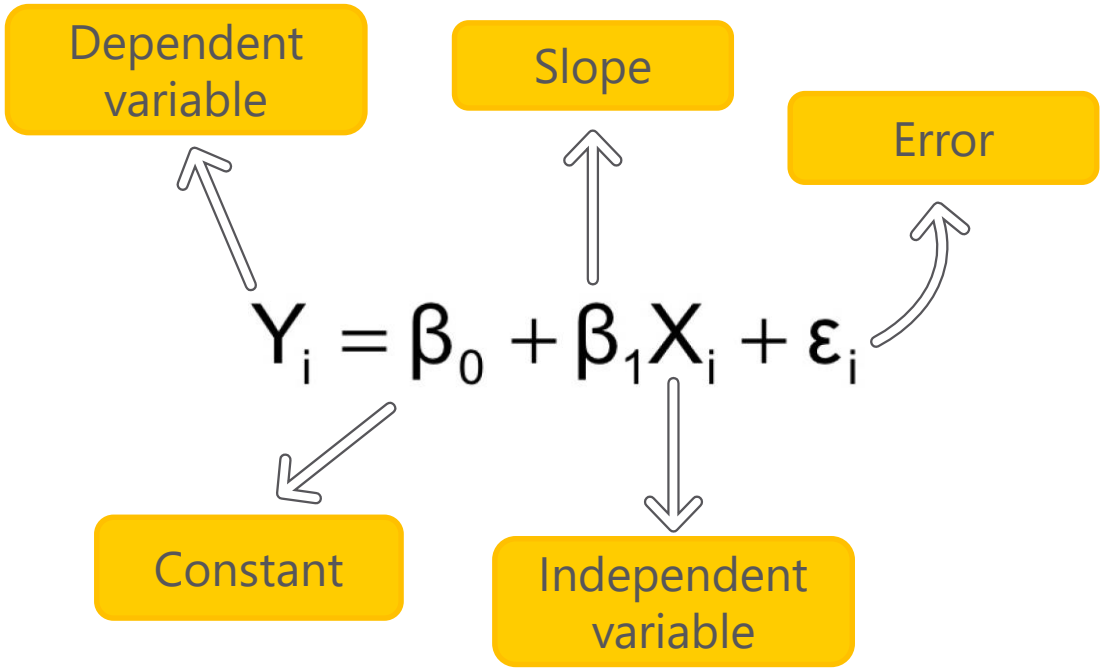
Regression analysis is one of the most widely used methods for prediction. Linear regression is probably the most fundamental machine learning method out there and a starting point for the advanced analytical learning path of every aspiring data scientist.

A linear regression is a linear approximation of a causal relationship between two or more variables.

Regression models are highly valuable, as they are one of the most common ways to make inferences and predictions. Apart from this, regression analysis is also employed to determine and assess factors that affect a certain outcome in a meaningful way.

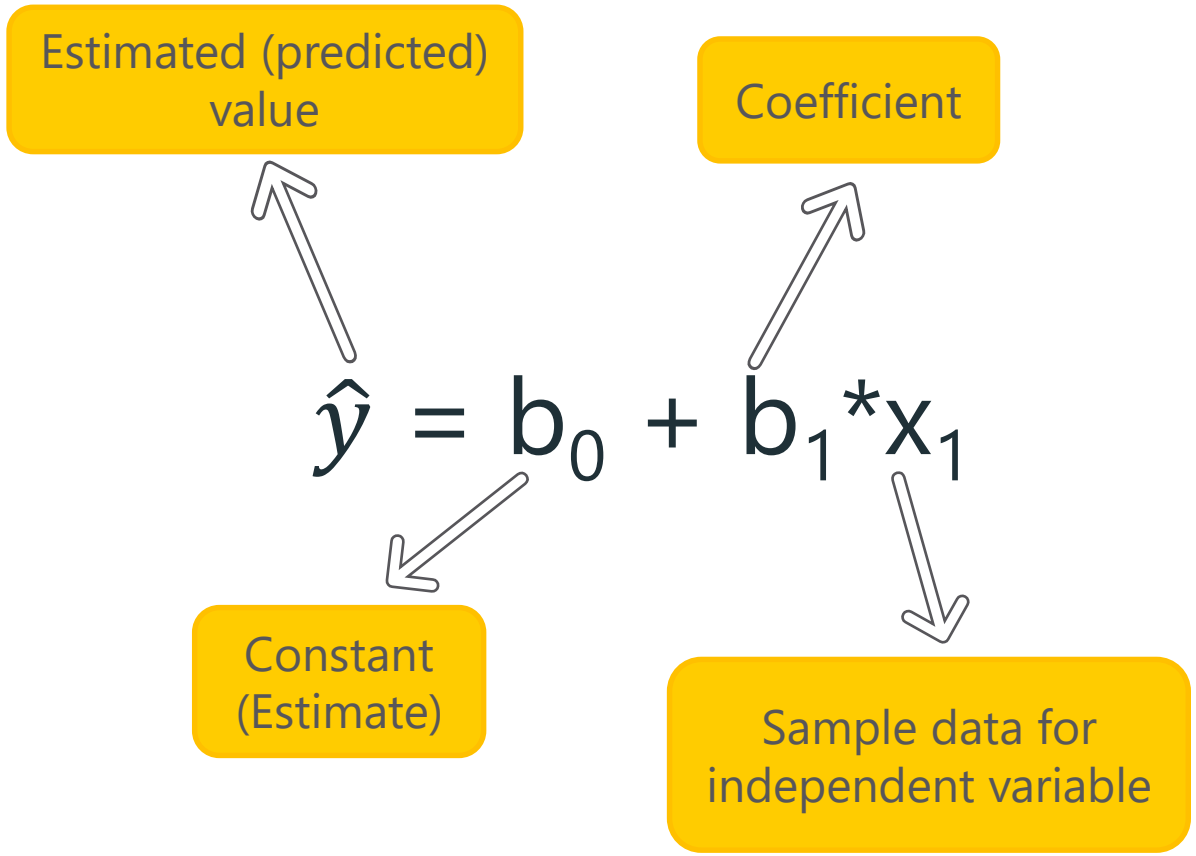
As many other statistical techniques, regression models help us make predictions about the population based on sample data.

Linear regression model



Note: When we refer to the population models, we use **Greek letters**

Linear regression equation



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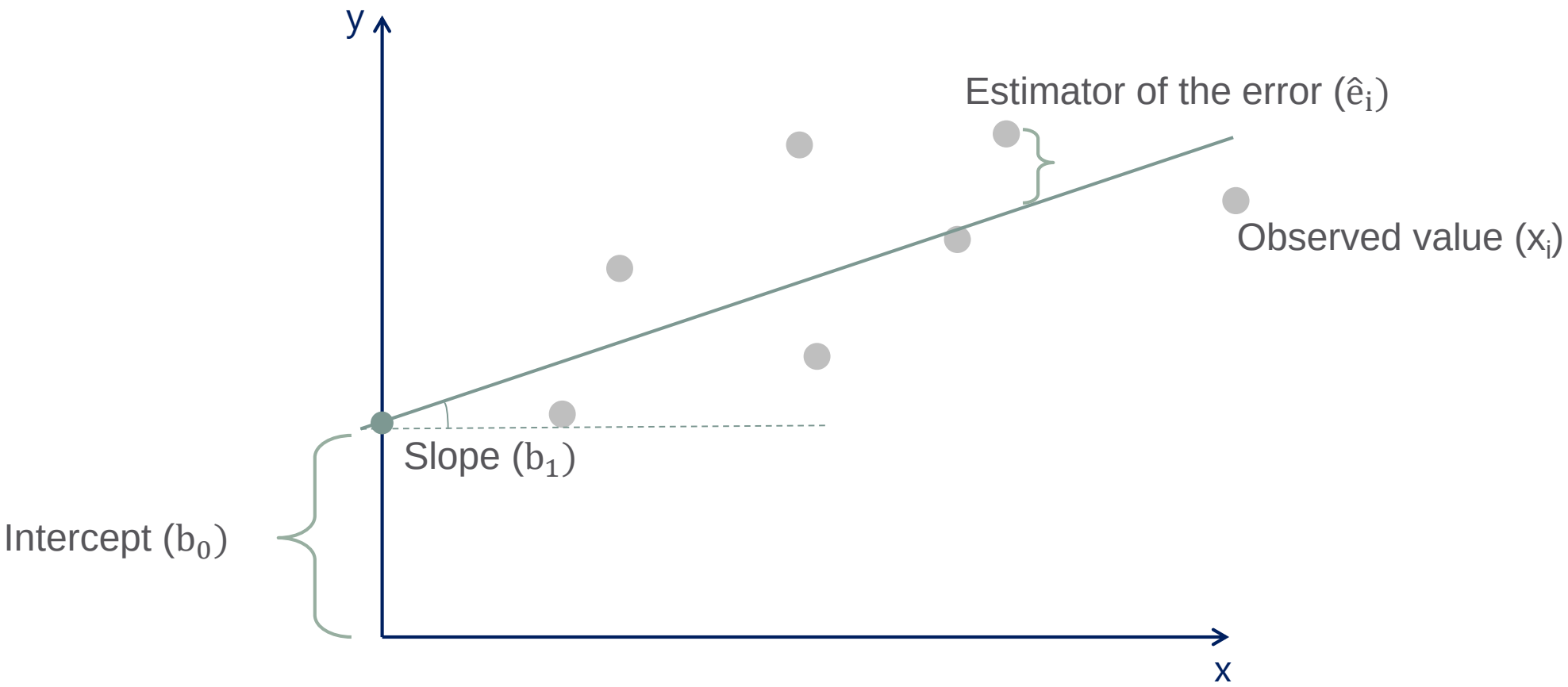
Geometrical representation of linear regression

$$\hat{y}_i = b_0 + b_1 x_i$$



*On average the expected value of the error is 0, that is why it is not included in the regression equation

Correlation	vs	Regression
Represents the relationship between two variables		Represents the relationship between two or more variables
Shows that two variables move together (no matter in which direction)		Shows cause and effect (one variable is affected by the other)
Symmetrical w.r.t. the two variables: $\rho(x,y) = \rho(y,x)$		One way – there is always only one variable that is causally dependent
A single point (a number)		A line (in 2D space)

Summary table and important regression metrics

Variability of the data, explained by the regression model
Range: [0;1]

Variability of the data, explained by the regression model, **considering the number of independent variables**
Range: <1; could be negative, but a negative number is interpreted as 0

The dependent variable, y; This is the variable we are trying to predict

Dep. Variable:	GPA	**R-squared:**	0.406
Model:	OLS	**Adj. R-squared:**	0.399
Method:	Least Squares	**F-statistic:**	56.05
Date:	Fri, 22 Nov 2019	**Prob (F-statistic):**	7.20e-11
Time:	15:29:11	**Log-Likelihood:**	12.672
No. Observations:	84	**AIC:**	-21.34
Df Residuals:	82	**BIC:**	-16.48
Df Model:	1		
Covariance Type:	nonrobust		

P-value for F-statistic; F-statistic evaluates the overall significance of the model (if at least 1 predictor is significant, F-statistic is also significant)

Coefficient of the intercept, b_0; sometimes we refer to this variable as constant or bias (as it 'corrects' the regression equation with a constant value)

	coef	std err	t	P>|t|	[0.025	0.975]
const	0.2750	0.409	0.673	0.503	-0.538	1.088
SAT	0.0017	0.000	7.487	0.000	0.001	0.002

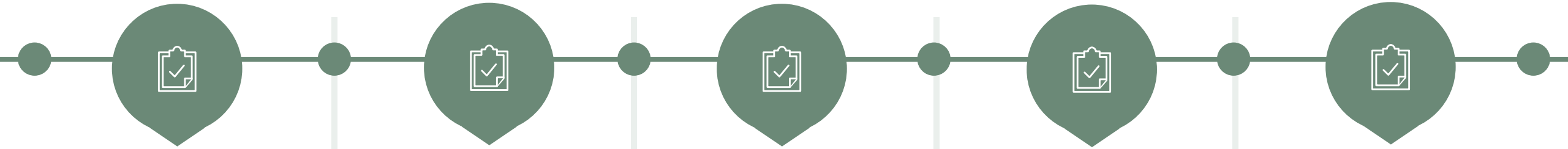
P-value of t-statistic; The t-statistic of a coefficient shows if the corresponding independent variable is significant or not

Coefficient of the independent variable i: b_i; this is usually the most important metric – it shows us the relative/absolute contribution of each independent variable of our model

Omnibus:	12.839	**Durbin-Watson:**	0.950
Prob(Omnibus):	0.002	**Jarque-Bera (JB):**	16.155
Skew:	-0.722	**Prob(JB):**	0.000310
Kurtosis:	4.590	**Cond. No.**	3.29e+04

OLS assumptions

OLS (ordinary least squares) is one of the most common methods for estimating the linear regression equation. However, its simplicity implies that it cannot be always used. Therefore, all OLS regression assumptions should be met before we can rely on this method of estimation.



Linearity

$Y=\beta_0+\beta_1 X_1+\ldots+\beta_k X_k+\varepsilon$

The specified model must represent a linear relationship

No endogeneity

$\sigma_{X\varepsilon} = 0 : \forall\, x, \varepsilon$

The independent variables shouldn't be correlated with the error term.

Normality and homoscedasticity

$\varepsilon \sim N(0, \sigma^2)$

The variance of the errors should be consistent across observations.

No autocorrelation

$\sigma_{\varepsilon_i \varepsilon_j} = 0 : \forall\, i \neq j$

No identifiable relationship should exist between the values of the error term.

No multicollinearity

$\rho_{x_i x_j} \approx 1 : \forall\, i, j;\ i \neq j$

No predictor variable should be perfectly (or almost perfectly) explained by the other predictors.

Other methods for finding the regression line

OLS (ordinary least squares) is just the beginning. OLS is the simplest, although often sufficient method to estimate the regression line. In fact, there are more complex methods that are more appropriate for certain datasets and problems.